Dispersion Engineering and Low-Loss Optimization of Footprint-Efficient and Rotationally Asymmetric Resonators

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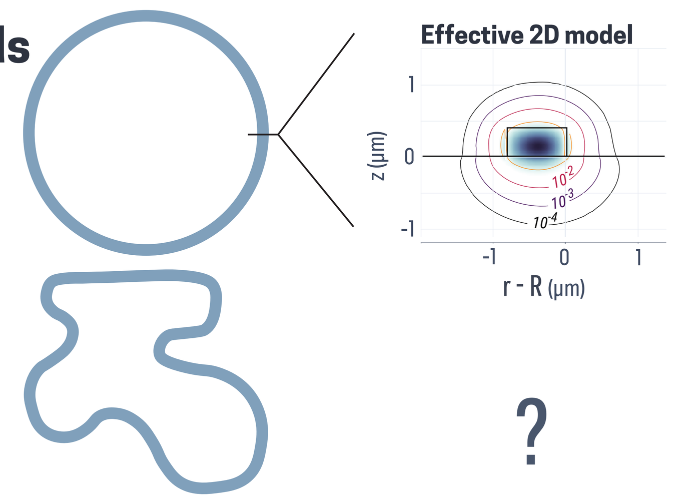
Universal Blueprint for Intuitive Resonator Design

The Limitations of Integration

The large footprint of low free spectral range (FSR) resonators has limited the integration of systems for broadband technologies such as LiDAR, microwave synthesis, and 5G, which require GHz repetition rates to interface with the microwave domain. FSR depends on both group index and resonator path length. However, since high n_g usually introduces high losses, **it is more straightforward to maximize the resonator path length per chip surface area.** To this end, many have explored wrapping waveguides over very small areas on chips, leading to radially asymmetric resonators.

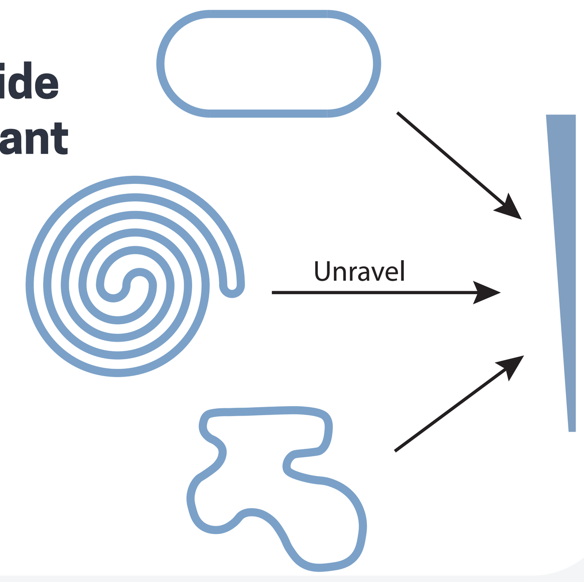
Challenges of Waveguide Bends with Non-Constant Curvature

Microring resonators are one of the most common photonics cavities as they possess a convenient rotational symmetry that allows them to be represented by the effective 2D model, for computationally-efficient simulations. By writing E(r, θ, z) = A(r, z)exp(imθ), one need only solve for A(r, z). For structures with non-constant curvatures, however, there is currently **no effective way to capture their dynamics aside from full 3D simulations**, which are very time consuming.



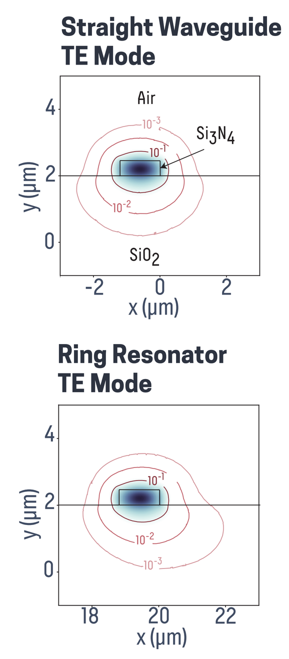
Challenges of Waveguide Bends with Non-Constant Curvature

We develop a framework for **unraveling any arbitrary bend into an effective straight waveguide** system with effective geometric and material properties, opening the door for intuitive design and simulation of previously inaccessible resonator structures.



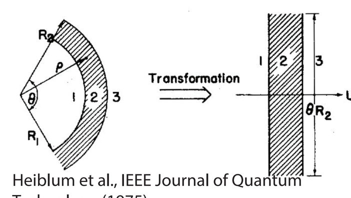
A Tale of Two Transformations

Since waveguides guide light at the scale comparable to the wavelength of light, both the geometry and material composition of the system affect mode properties such as effective index and effective area. Introducing such geometric bending like that in rings shifts the mode profile toward the outer edge of the waveguide. **Conformal mapping** and **transformation optics** are two approaches used to model these changes in mode properties.



Conformal Mapping

$$u + iw = R\ln\frac{x+iz}{R} \qquad v = y \quad \rightarrow \quad u = R\ln\frac{r}{R} \qquad w = R\theta$$



Heiblum et al., IEEE Journal of Quantum Technology (1975)

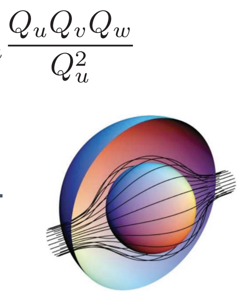
Differential Conformal Transformation Optics (DCTO)

Transformation Optics

$$\varepsilon'_u = \varepsilon_u\frac{Q_uQ_vQ_w}{Q_u^2} \qquad \mu'_u = \mu_u\frac{Q_uQ_vQ_w}{Q_u^2}$$

where

$$Q_i^2 = \left(\frac{\partial x}{\partial i}\right)^2 + \left(\frac{\partial y}{\partial i}\right)^2 + \left(\frac{\partial z}{\partial i}\right)^2$$



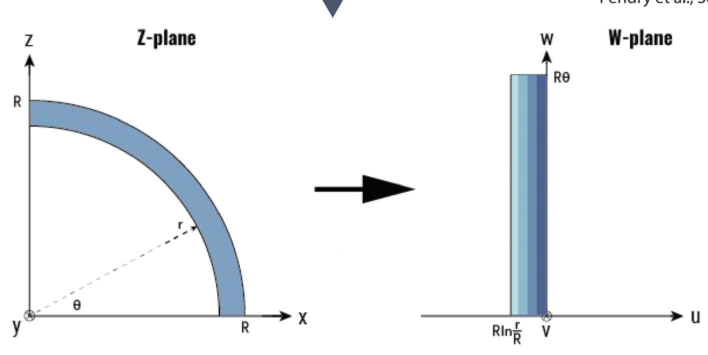
Pendry et al., Science 312, 1780 (2006)

For the previously defined conformal mapping,

$$Q_u = Q_w = e^{u/R} \qquad Q_v = 1$$

$$\begin{bmatrix}\varepsilon'_u\\ \varepsilon'_v\\ \varepsilon'_w\end{bmatrix} = \begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & e^{2u/R} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\mu'_u\\ \mu'_v\\ \mu'_w\end{bmatrix} = \begin{bmatrix}\mu_x\\ \mu_y\\ \mu_z\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & e^{2u/R} & 0\\ 0 & 0 & 1\end{bmatrix}$$



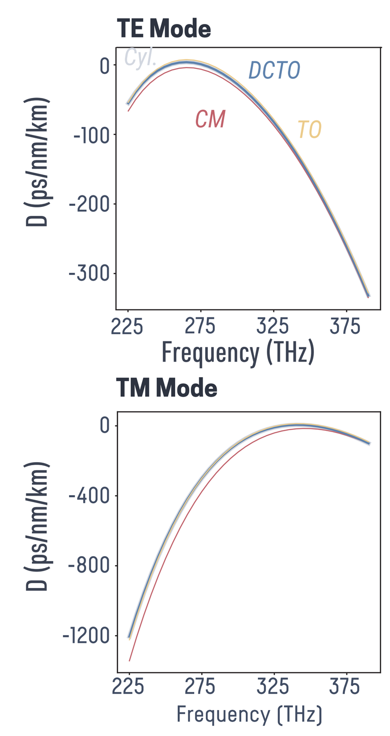
Aside: One consequence of the synthetic anisotropy is that **the transformation behaves differently depending on the polarization of the mode.** For TE (TM) modes, there exists no electric field in the direction of propagation, or equivalently, Ey (Hy) can be approximated as null. Therefore, only the anisotropy in μ (ε) affects the mode in the equivalent straight waveguide.

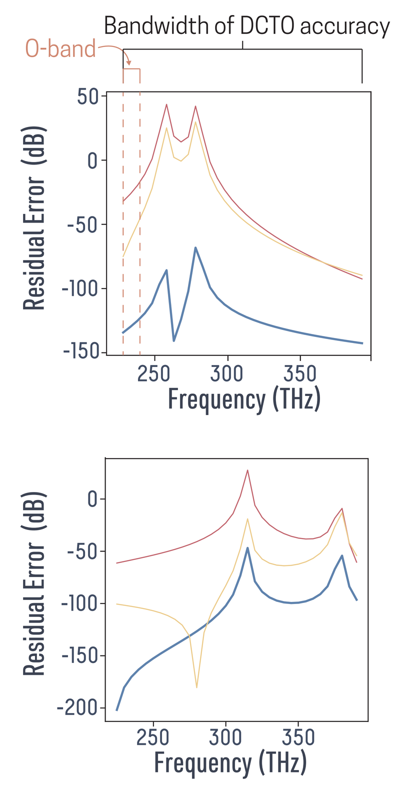
Validating DCTO

We simulated the dispersion of **Si3N4 microring resonators** using the effective straight waveguide systems predicted by DCTO, conformal mapping (CM), and transformation optics (TO), and compared them against a commercially available cylindrical model. DCTO consistently produced more accurate results compared to accurate, fully-vectorial cylindrical simulations than both CM and TO. **This accuracy holds over a >150 THz wide bandwidth and regardless of ring geometry.**

Dispersion equation

$$D = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$





DCTO Design Guidelines for Low-Loss Racetracks

DCTO Applied

The bend of a racetrack can be segmented into infinitesimally small path lengths to apply the transformation differentially, as for each differential path length ds, an instantaneous R(s) can be defined. Effective waveguide width h' will change depending on original width h and as a function of path length.

$$h'(s) = |R(s)\ln\frac{R(s)-h}{R(s)}|$$

DCTO Equations

$$u = R(s)\ln\frac{r}{R(s)}$$

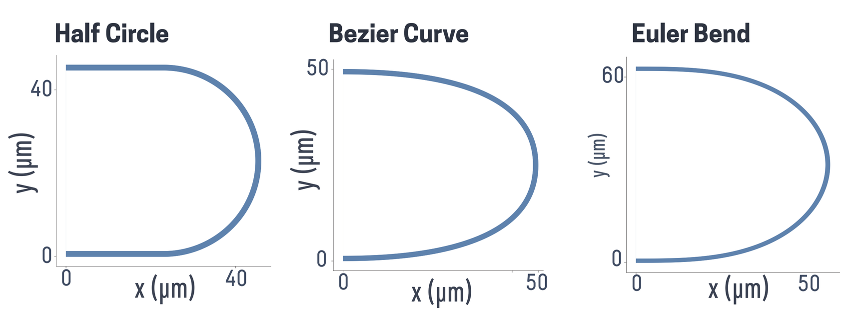
$$dw = ds$$

DCTO maps any bend into a straight waveguide with varying width and effective material properties. In this more intuitive space, it is easy to visualize why certain bends are advantageous over others for mitigating loss.

$$P_{1,2} \propto \int_S \varepsilon E_1 \cdot E_2^* \, \mathrm{d}S$$

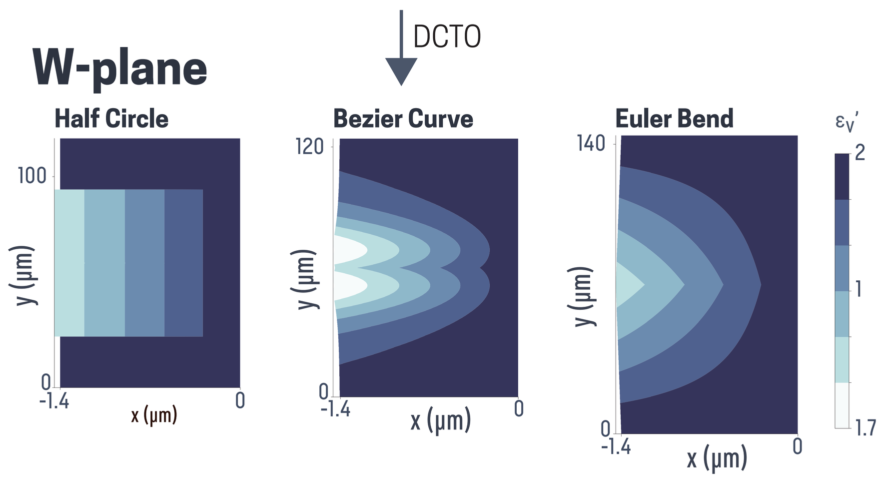
Losses occur when the mode overlap in the propagation direction is less than 1. Thus, **continuity of effective geometry and εv in the propagation direction**, which determine the placement of the mode profile, is both crucial and straightforward to engineer with DCTO.

Z-plane



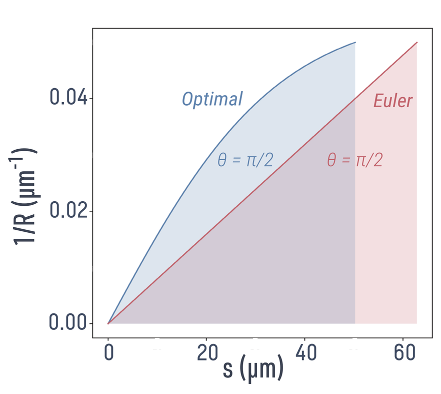
DCTO

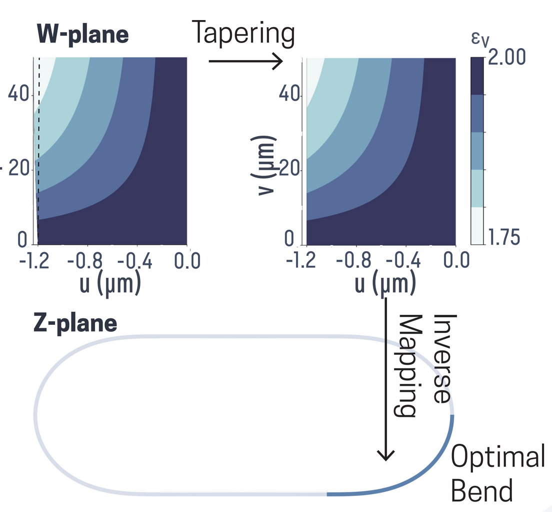
W-plane



Optimal Low-Loss Bend Design

DCTO allows for intuitive first-generation design of low-loss, numerical bends prior to simulation through simply optimizing for smooth effective geometry and εv in the virtual space and then applying the inverse transformation.

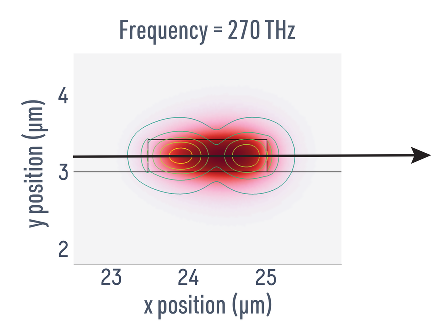


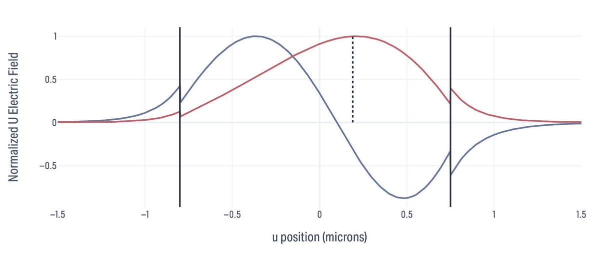


DCTO Design Guidelines for Eliminating Avoided Mode Crossings (AMX)

Quantifying AMX

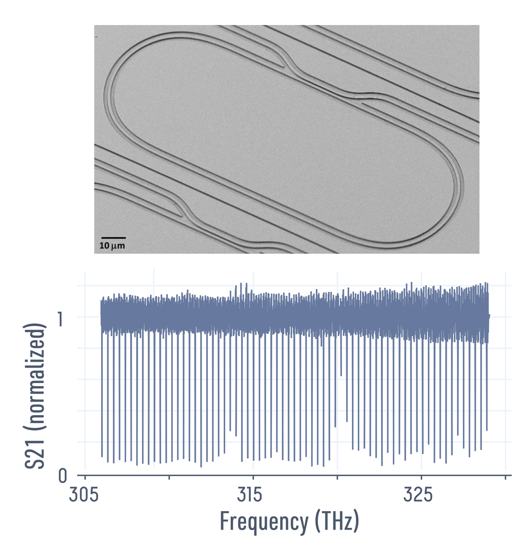
The non-orthogonality of higher order modes is another consequence of shifted mode profiles caused by geometric bending. This results in a non-zero overlap between these modes, **allowing them to couple and transfer power**.

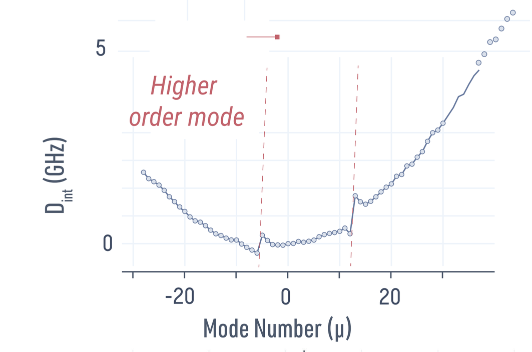


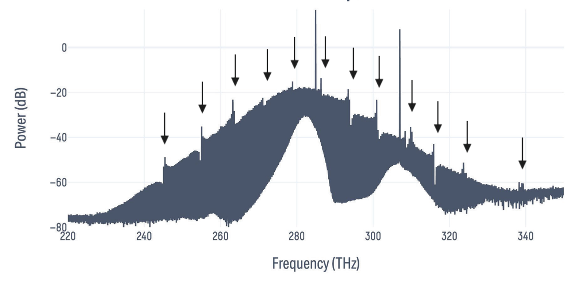


Experimental Observation

AMX were observed experimentally in Euler bend racetracks. A coupling between the fundamental and higher order TE mode could occur if they are close enough in frequency and propagation constant, **resulting in a hybridization**. The coupling will lift the degeneracy of the frequencies of resonance, **splitting them in two distinct mode frequencies** that are not aligned with the FSR variation of the remainder of their respective mode families.

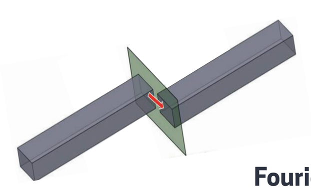






Coupled Mode Theory (CMT) Formalism

In progress:
A CMT formalism for predicting energy transfer between two modes in the same waveguide.

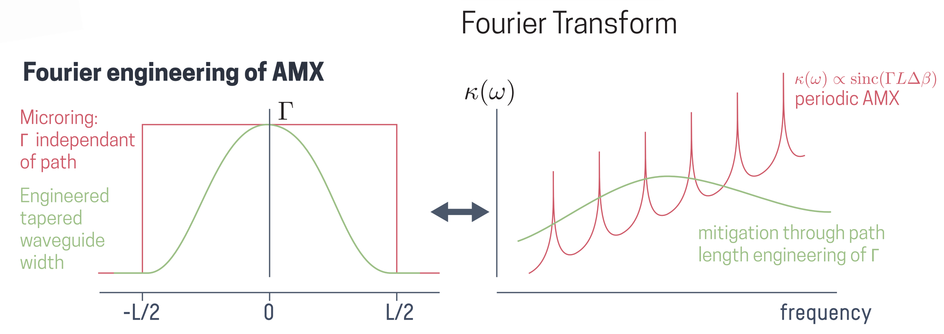


$$\kappa(\omega) = \frac{i\omega}{\sqrt{2P_{TE0}P_{TE1}}}\int_L \Gamma\exp(i\Delta\beta l)\mathrm{d}l$$

Overlap function

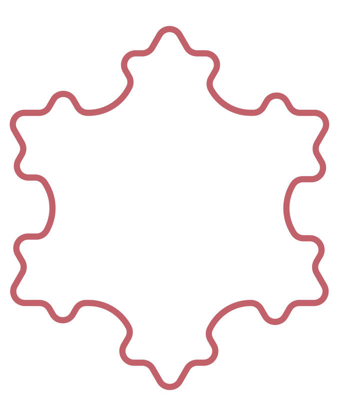
Fourier Transform

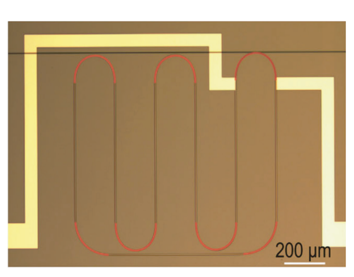
DCTO unravels not only the geometry of the bend but also the k vector of the mode, allowing it to remain collinear throughout the structure, greatly simplifying the CMT formalism. Butt-coupling assumes two modes with a non unitary overlap due to positional mismatch. We can integrate this formalism in our DCTO waveguide to obtain the power transfer over one round trip from the fundamental TE to the higher order TE modes.



Arbitrary Bending Optimization

We have presented an application of DCTO to racetrack resonator design, however **this exact method can be applied just as easily for far more complicated geometries**. Racetracks are not the most footprint-efficient resonators—beyond racetracks there are spiral and fractal-based designs. Although some work has been done in optimizing racetracks, going any further would require highly computationally intensive and time consuming simulations. With DCTO, **most of the optimization can be done before any simulation or computation is necessary**.





Ye et al., Laser Photonics Rev. 16, 2100147 (2022)

Global circular geometry (racetrack, spiral, etc..) → DCTO to obtain W plane → Anisotropy apodization in W-plane for low losses → Taperization of waveguide in W-plane to prevent avoided-mode crossing → Reverse DCTO to obtain real Z-plane

Acknowledgements

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Chip fabricated at LIGENTEC

For more information, check out our lab webpage